Effects of Charged Particles on the Motion of an Earth Satellite

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Using a model of the upper atmosphere in which the principal atmospheric constituent (for both atoms and ions) changes with increasing altitude from atomic oxygen to helium and then to atomic hydrogen, the charge on a conducting spherical satellite and the drag due to charged particles are estimated for altitudes below the hydrogen region. The effects of photoelectric emission and the earth's magnetic field are included in the calculations. It is found that the contribution of charged particles to the satellite drag may be neglected in the oxygen region. In the helium region the drag due to charged particles may be significant, particularly for large satellites and in cases where the photoelectric emission current is significant. A method of estimating this drag is presented. A comprehensive survey of the literature concerning satellite charge and the resulting drag is included, and the existing contradictions are explained. The conclusions regarding drag are applicable to satellites large with respect to the debye length.

THE prediction of satellite decay times, as well as the use of measured rates of decay to gain information about the properties of the upper atmosphere, requires accurate knowledge of the forces acting on a satellite. These forces arise through the interaction of the satellite with the charged and neutral particles in the atmosphere. While the effects of neutral particles are reasonably well understood, in recent years a number of papers have been published presenting more or less contradictory predictions of the drag due to charged particles. In this article a thorough review and analysis of the published work on this problem is presented, and equations are developed whereby the upper limits of the drag due to charged particles can be estimated. The causes of the contradictions in predicted charged particle drag are explained, and an over-all description of the phenomena involved is given. While it will be seen that certain aspects of the problem cannot be definitely resolved, the drawing of speculative conclusions has been avoided.

At usual satellite altitudes ($h > 300$ km) the mean free paths for collisions between neutral particles are in excess of 1 km, and hence any practically conceivable satellite falls well within the free molecule flow regime. The calculation of satellite drag due to collisions with neutral particles has been treated in detail by Schaaf[1] and Schamberg.[2] In addition to neutral particles, the upper atmosphere contains significant numbers of ions and electrons. According to the estimates of atmospheric data of Ref. 3, the atmosphere is about 0.1% ionized at 300 km, and the percentage of ionization rises with increasing altitude. At 4000 km essentially all gas molecules are ionized. It is felt that only singly charged positive ions are present above 300 km, and hence the usual requirement of plasma neutrality implies that equal numbers of ions and electrons are present at these altitudes ($n_e = n_i$). The mean free paths for charge-neutral collisions are of the same order of magnitude as those for neutral-collisions, and hence need not be considered further. However, since the interactions between charged particles are due to coulomb forces, which act over a long range, some care must be exercised in adopting a free molecule flow approach in the case of an ionized gas. The deflection of a charged particle is actually due to a large number of long range encounters, each of which alters the particle motion only slightly. The usual procedure is to define the collision time as the time necessary for a particle to be deflected through 90 degrees. If the body dimensions are kept much smaller than the mean free paths calculated on the basis of these collision times, the assumption of free molecule flow should be valid. The collision times for charged particle-charged particle collisions may be calculated using the equations given by Spitzer.[4] These equations give a value of electron-ion mean free path λ_{ei} somewhat smaller than that calculated by Nicolet and presented in Ref. 3, and thus represent a lower limit on the estimate of λ_{ei}.† The charged particle mean free paths increase with increasing temperature or decreasing number density. Using the data of Ref. 3, and the more recent estimates of upper atmosphere temperatures cited by Jastrow[6] to select the combination of density and temperature yielding a minimum, the minimum mean free path for electron-electron or ion-ion collisions is calculated to be $\lambda_{ee} = \lambda_{ii} \approx 100$ m. The corresponding estimate of electron-ion mean free path is $\lambda_{ei} \approx 70$ m. The mean free paths increase rapidly with increasing altitude, so except for extremely large satellites at very low altitudes the use of a free molecular flow approach should be valid.

The calculation of the drag on a satellite due to the presence of charged particles in the upper atmosphere is essentially the calculation of the drag on a charged body moving in an ionized gas (plasma) in the presence of a magnetic field. The drag may be considered as coming from three factors, although these factors may not be completely unrelated. First there is the drag caused by the deflection of ions and electrons by the charged body. These ions and electrons may strike the body, or they may simply be deflected due to coulomb forces. Second, there is the so-called "induced drag," produced by the current flow induced in a conductor which moves in a magnetic field. Third, there is the factor which might be called "wave drag," due to collective plasma motion induced by the movement of the charged body. The charging of the satellite will be considered first and then the methods of estimating the various drag forces will be discussed.

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† It should be noted that the mean free path calculated from the electron-ion collision frequency applies only to the electrons, since ions are essentially unaffected by such collisions. The extremely short ionic mean free paths calculated by Chopra[5] are the result of the incorrect use of the electron-ion collision frequency as the characteristic collision frequency for ions.

Table 1 Most recent estimates of conditions in the upper atmosphere for various values of altitude h^a

h, km	$\bar{c}_e$ m/sec, T = 500°K	$\bar{c}_i$ m/sec T = 500°K	$\bar{c}_i$ m/sec T = 2200°K	Principal atoms	v_s, m/sec	Neutral particles, n/m³	Charged particles, $n_e = n_i$/m³
300	1.39×10^5	0.79×10^3	1.65×10^3	O	7.7×10^3	2×10^{15}	2×10^{12}
500	1.39×10^5	0.81×10^3	1.7×10^3	O	7.6×10^3	2×10^{14}	2×10^{11}
1000	1.39×10^5	0.81×10^3	1.7×10^3	O	7.35×10^3	5×10^{12}	10^{10}
2000	1.39×10^5	1.6×10^3	3.4×10^3	He	6.9×10^3	10^{11}	5×10^9
3000	1.39×10^5	1.6×10^3	3.4×10^3	He	6.5×10^3	2×10^{10}	2×10^9
4000	1.39×10^5	3.2×10^3	6.8×10^3	H	6.2×10^3	10^9	2×10^9

^{a} Typical value of electron thermal velocity c_e, ion thermal velocity $\bar{c}_i$, circular orbit satellite velocity v_s, and particle densities are shown. Particle densities are mean values for $T \approx 1600$°K.

Satellite Potential Neglecting Photoelectric Emission

It will be useful to begin with a brief discussion of the behavior of a body in an ionized gas (with or without relative motion between the gas and the body). It is clear that collisions between the body and the ions and electrons will occur. Since it is not possible for an isolated body to sustain a net current flow to or from the plasma, the body must become charged in such a manner that the net fluxes of ions and electrons to the body are the same.‡ It is usual to assume that all collisions with the body result in neutralization of the colliding particles and that the body is made of a conducting material. Under these assumptions it is possible, at least in theory, to compute the equilibrium potential which the body will assume. A correction factor to allow for the reflection of charged particles as charged particles can be applied to this result.

On the basis of theoretical and experimental results,[7, 8] it is felt that the electron and ion temperatures in the upper atmosphere are equal and that all particles have velocity distributions which are very nearly Maxwellian. Thus the mean thermal velocity of electrons $\bar{c}_e = (8kT/\pi m_e)^{1/2}$ (where k = Boltzmann's constant and m_e = electronic mass) is about 43 times as great as that of protons $\bar{c}_i = (8kT/\pi m_i)^{1/2}$. If the ions are heavier particles than protons, or if the electron temperature exceeds the ion temperature, the relative magnitude of $\bar{c}_e$ will be still greater. Thus it is clear that, in the case of a stationary body, since $n_e = n_i$ and $\bar{c}_e \gg \bar{c}_i$ the net flux of electrons incident on a unit surface area of an unshielded body is much greater than the net flux of ions, i.e.; $(n_e\bar{c}_e/4) \gg (n_i\bar{c}_i/4)$. Under steady conditions no net current can flow to or from the body, so the body must take on a negative potential such that all but a number of high energy electrons sufficient to balance the ion current are repelled. The equality of electron and ion currents can thus be expressed as

$$-I_i = \frac{en_i\bar{c}_iA_i}{4} = I_e = \frac{en_e\bar{c}_eA_e}{4}\, e^{-e\varphi_s/kT} \qquad (1)$$

where φ_s is the potential of the body, e the electronic charge, defined negative, and A_e and A_i the effective collection areas for electrons and ions. The equilibrium potential on a stationary body can be calculated from Eq. (1). The effective collection area A_i may be a function of potential, and the surface area of the body represents a lower limit on A_i. If the body is in motion, the ion and electron fluxes will depend on the velocity of the body, and hence the equilibrium potential is not so easily obtained. It should also be noted that the bombardment of the body by photons may cause the photoemission of electrons, with a resultant change in the body potential in the positive direction. This effect will be considered later.

‡ It should be noted that the quantity which can be readily computed is not the charge, but the potential which the body assumes. This potential is referred to the so-called plasma potential, which is the potential at which the body collects all the ions and all the electrons incident upon it.

Satellite potentials have been calculated by a number of authors. While Chopra[5] considers only the stationary case just given, all other published results[9–14] assume that $\bar{c}_i \ll v_s \ll \bar{c}_e$ where v_s is the satellite velocity. Thus, in a frame of reference moving with the satellite, the thermal motion of the ions is regarded as negligible compared to their directed motion, whereas the directed motion of the electrons is negligible compared to their thermal motion. It is of interest to consider the extent to which the forementioned approximations are satisfied in the upper atmosphere. Table 1 has been calculated on the basis of the most recently published estimates of the properties of the atmosphere at altitudes greater than 300 km.[3, 6, 15] The satellite velocities are based on circular orbits. It should be noted that considerable seasonal variations in density and temperature, corresponding to variations in solar activity, can occur, as well as diurnal variations. The daily variation in temperature may be as high as 800°K, and the upper and lower limits of temperature are estimated as 2200° and 500°K; the upper limit corresponding to an afternoon temperature at maximum solar activity and the lower limit to a predawn temperature at minimum activity. These temperature variations are accompanied by density variations which can amount to several orders of magnitude. The densities quoted correspond to a temperature of 1600°K. As one moves outward from the earth, the principal constituent of the atmosphere changes from atomic oxygen at 3000 km, to helium at about 1000 km, and finally to hydrogen at around 2500–3500 km. The transition points are subject to the same variations mentioned earlier.

It can be seen from Table 1 that $v_s \ll \bar{c}_e$ is always well satisfied, whereas $v_s \gg \bar{c}_i$ becomes an increasingly poor approximation with increasing altitude or increasing temperature. The neglect of the ionic thermal velocity is certainly open to question wherever the principal atmospheric constituent is hydrogen.

The limiting values of the satellite potential will be considered first and then the results will be compared with those published results mentioned earlier. The treatment will be limited to the case where $\bar{c}_i \ll v_s \ll \bar{c}_e$, and thus is not applicable to the hydrogen region. The effects of photoelectric emission and the earth's magnetic field will be introduced later. If the effect of satellite potential on the ions is neglected, the effective collection area for ions is simply the projected area of the sphere. Hence the incident ion flux is $n_iv_s\pi R^2$, where R is the sphere radius. This is identical to the result obtained for neutral particles, since in effect it is assumed that the satellite potential is screened in a negligibly thin region outside the surface.

In the opposite case, in which the satellite potential is assumed to be completely unshielded, use may be made of the results first given by Mott-Smith and Langmuir[16] with reference to probe theory in gaseous discharges. From a consideration of the conservation of angular momentum and the conservation of energy of a singly charged particle moving in the vicinity of a charged sphere, the effective impact radius for ions is calculated as $b_i = R[1 + (2e\varphi_s/m_i v_s^2)]^{1/2}$.

The electron current to the satellite is dependent only on the satellite potential, and not on the degree of shielding outside the satellite surface, since only those electrons with energies greater than $e\varphi_s$ can reach the satellite surface and recombine with ions. Thus the incident electron flux is as given in Eq. (1), provided a suitable value of A_e is used. Since the thermal velocity of the ions is small compared to the satellite velocity, a region of reduced ion density may exist behind the satellite. If over-all charge neutrality is maintained in this region (as will be seen to be the case) the electron density will also be reduced. Thus the effective collection area for electrons must be modified to account for the wake. The limiting values can be bracketed by $2\pi R^2 \leq A_e \leq 4\pi R^2$. The electron and ion currents to a spherical satellite are given by

$$\left.\begin{aligned} I_i &= -en_i v_s \pi R^2 C_i \\ I_e &= en_e 4\pi R^2 (kT/2\pi m_e)^{1/2} e^{-(e\varphi_s/kT)}\, C_e \end{aligned}\right\} \qquad (2)$$

where $C_i = 1$ for complete shielding, $C_i = 1 + (2e\varphi_s/m_i v_s^2)$ for no shielding, $C_e = 1$ for no wake correction, and $C_e = \frac{1}{2}$ for wake on rear half. It will be seen that the effect of ion attraction by the negatively charged satellite is to reduce the wake region, and that for most cases partial shielding exists. However, Eq. (2) is felt to represent the limiting behavior correctly.

The satellite potential is determined by equating the electron and ion currents in Eq. (2), as

$$\varphi_s = \frac{kT}{2e}\left[\ln\left(\frac{8kT}{\pi m_e v_s^2}\right) - 2\ln\frac{c_i}{c_e}\right] \qquad (3)$$

The second term in brackets is always much smaller than the first, and below the transition point to hydrogen the maximum correction amounts to less than 25%. Below the helium region the uncertainty is still smaller. The maximum negative potential consistent with Table 1 is $\varphi_s \approx -0.75$ v.

In connection with the forementioned results, it should be noted that the use of conservation of angular momentum about the sphere center is only justified when the charge distribution about the sphere is spherically symmetric. It has been shown by Heatley[17] in connection with probe theory, and can also be demonstrated by simple arguments concerning the effect of a negative charge in the wake on the angular momentum of a particle, (because of the deficiency of ions in the wake, a negative charge sufficient to reduce the electron density and maintain charge neutrality is built up) that the neglect of the lack of spherical symmetry in the charge distribution causes both the electron and ion currents to be overestimated. However, in view of the uncertainty which exists in the effective collection area for electrons, and in view of the fact that the inclusion of ion attraction yielded only a small correction term in Eq. (3), a more exact calculation does not appear to be necessary.

One of the first estimates of satellite potential is that of Jastrow and Pearse.[9] In the nomenclature of Eqs. (2) and (3), they use $C_i = 1$, $C_e = 1$. They also assume that $T_e \gg T_i$, and on this basis predict negative potentials up to $\varphi_s = -60$ v. The later experimental and theoretical results indicate that $T_e \approx T_i$, and hence such high potentials would not be expected to occur.

In contrast to Ref. 9, Chang and Smith[11] assume that the completely unshielded results are valid, and also introduce a wake over the rear hemisphere. The effect of photoelectric emission on the satellite potential is also considered, and it is concluded that for lower altitudes ($k < 1000$ km) the photoelectric effect is small. By means of an approximate analysis of the potential distribution around the satellite, Chang and Smith attempt to show that the unshielded solutions are correct. For the spherically symmetric problem, the maximum impact parameter which an ion may have and still reach any given radius r from the sphere center is $b = r\{1 + [2e\varphi(r)/m_i v_s^2]\}^{1/2}$. For the unshielded solutions to be correct, there must exist no $r > R$ for which $b(r) < b_i(R)$. Thus it must be shown that $db/dr \geq 0$ for $r \geq R$. There appears to be an error in sign in Chang and Smith's analysis, and a recalculation shows that, for the approximate potential distributions calculated by them, appreciable shielding does occur at higher electron densities and lower potentials. The whole question of the charge distribution around the satellite will be discussed in more detail in a later section.

Beard and Johnson[10] modify Jastrow and Pearse's approach to include the effect of the earth's magnetic field. The case of a rectangular parallelepiped moving at right angles to the field lines is considered, and it is assumed that the mean motion of the electrons is parallel to the magnetic field, so that electrons are incident only on the two surfaces perpendicular to the field direction. As before, the ion current is determined by the projected area in the direction of motion of the body.

Due to the presence of a magnetic field, a potential gradient of Bv_s volts/m in the $\mathbf{v}_s \times \mathbf{B}$ direction is induced in the satellite, where B is the magnetic field strength in webers/m^2 and v_s the satellite velocity in m/sec. The effect of this potential gradient is to concentrate the electron current toward the positive end of the satellite. The ion current will be unaffected, except insofar as the increased negative potential tends to increase the effective collection area for ions. The magnetic field strength varies considerably with geographical location (see Ref. 3 for a more complete discussion) but is always of the order of 10^{-5} webers/m^2 at altitudes of interest. To summarize briefly the results presented in Ref. 10, it can be stated that for satellites whose length is less than say 1 meter the magnetic field effects will be small, causing only a small change in the average (midpoint) satellite potential. As the satellite length increases, the effect of the potential gradient increases until for very large satellites (say length > 10 m) one end of the satellite is essentially at the plasma potential, and the average potential is given by $\varphi = Bv_s l/2$ where l is the total length in meters. Magnetic field effects will be neglected in the remainder of this section, hence we effectively consider only small satellites or those moving parallel to the field lines.

In a paper published at the same time as Ref. 9, Gringauz and Zelikman[12] discuss briefly the general question of satellite potentials. The existence of a potential gradient is pointed out, and the effective collection area for electrons is taken as the projected area of the satellite in a plane normal to the magnetic field lines. The necessity of calculating an effective collection area for ions is mentioned, but no method of determination is given. As a rough estimate, and without details of the method of calculation, negative potentials of less than 1 v are predicted for $T_e = T_i = 1000$°K.

For completeness, the pioneering paper by Lehnert[13] should be mentioned. The general features of the satellite potential problem are pointed out, and results similar to those of Ref. 12 are presented. This paper represents the first published treatment of the problem.

Satellite Potential Including Photoelectric Emission

As will be seen, the experimental results indicate that the effect of photoelectric emission is not negligible, and it will be convenient to modify Eq. (3) to include this effect. Consider first the case where the resulting satellite potential is not greater than zero. Here the various relationships for the current flow to the satellite previously considered are still valid, and it is only necessary to rewrite the current balance to include the total photoelectric emission current I_{ph_m} which at this time must be regarded as a measured quantity.

Neglecting magnetic field effects, the satellite potential is given by

$$\varphi_s = \frac{kT}{2e}\left\{\ln\left(\frac{8kT}{\pi m_e v_s^2}\right) - 2\ln\left[C_i - \frac{I_{ph_m}}{v_s \pi R^2 n_e e}\right] + 2\ln C_e\right\} \tag{4a}$$

where the notation is that of Eq. (2).

The case of positive satellite potentials is more difficult; however, the limiting behavior can be discussed. Consider first the expressions for the current flow to the satellite. This problem has been treated in detail by Mott-Smith and Langmuir,[15] and only the relevant results will be cited. The ion current is given by

$$I_i = -\pi R^2 n_i e v_s \left(1 + \frac{2e\varphi_s}{m_i v_s^2}\right)$$

as before, as long as $\bar{c} \ll v_s$, except that for $(2e\varphi_s/m_i v_s^2) \leq -1$, $I_i = 0$. This expression is now valid regardless of any shielding of the satellite potential. The upper limit on the electron current occurs when no shielding is present, and is then

$$I_e = \pi R^2 n_e e \left(\frac{8kT}{\pi m_e}\right)^{1/2}\left(1 - \frac{e\varphi_s}{kT}\right)$$

If the sphere potential is shielded to some extent, the value of I_e may be reduced, and the dependence on φ will no longer be linear. The exact nature of the charge distribution around the body must then be known.

The photoelectric emission current also depends on the magnitude of the positive sphere potential φ_s. The measured value I_{ph_m} which was used in Eq. (4a) may be regarded as the maximum value, since the photoelectric emission does not increase with increasing negative potentials, the current being limited by the surface work function φ_w, and all emitted electrons being repelled from the surface. However, as the sphere potential becomes more positive, only the higher energy electrons can escape from the surface, and at any altitude a balance will be reached between the incident electron flux I_e and the photoelectric emission current I_{ph} at some positive potential. Thus one must estimate the dependence of the photoelectric emission current on potential. Since we are interested in the maximum positive potential which the satellite may assume, we will consider the upper limit of I_{ph}.

Thus assume that all electrons emitted by incident radiation of a certain frequency have the maximum energy associated with that frequency, as given by Einstein's equation $\frac{1}{2}m_e v_e^2 = h\nu + e\varphi_w$ (as before, e is defined negative), where ν is the frequency and h is Planck's constant. The simplest procedure is to assume that the sun's radiation can be approximated by a characteristic black body. The dependence of radiant energy on frequency is now given by Planck's formula, and, with an assumption regarding the efficiency of photoemission as a function of frequency, the dependence of I_{ph} on potential can be calculated. However, the data presented in Ref. 3 show that, while the incident radiation is reasonably well approximated by 4500°K black body up to a frequency of 2.1×10^{15} cps, the major portion of the solar radiation energy at higher frequencies is contained in a series of spectral lines, and the incident radiation per unit frequency no longer exhibits the exponential decay characteristic of a black body. If the Lyman α line at 1216 Å (2.46×10^{15} cps) is excluded, the remaining spectrum does not show any extremely pronounced peaks, and the energy per unit frequency varies roughly as the inverse square of the frequency. The Lyman α line contains about twice the energy contained in the remainder of the spectrum above 2.1×10^{15} cps. The energy levels are[3] $E = 0.25 \times 10^{-2}$ w/m² for $\nu > 2.14 \times 10^{15}$, plus $E = 0.6 \times 10^{-2}$ w/m² for $\nu = 2.46 \times 10^{15}$ (L_α). On the basis of this information, the incident solar energy can be represented by the following set of equations:

$$dE = 0.54 \times 10^{13}\,\frac{d\nu}{\nu^2} \qquad \nu > 2.14 \times 10^{15}$$

$$E = 0.6 \times 10^{-2} \qquad \nu = 2.46 \times 10^{15}$$

$$dE = 1.0 \times 10^{-54}\,\nu^3 e^{-(h\nu/kT_B)} \qquad \nu \leq 2.14 \times 10^{15}$$

where $T_B = 4500$°K is the effective black body temperature of the sun, and where the expression for $\nu \leq 2.14 \times 10^{15}$ makes use of the fact that $e^{-(h\nu/kT_B)} \ll 1$ for all frequencies above the threshold frequency of typical surfaces.

An accurate estimate of the dependence of the photoelectric emission on frequency is more difficult. The spectral yield is strongly dependent on surface condition and surface history, the uncertainty frequently amounting to more than a factor of ten. From the data given by Weissler[18] for typical surfaces, the dependence is roughly as follows. The photoelectric emission current per unit frequency for constant incident energy per unit frequency increases exponentially from the threshold frequency up to about 3×10^{15} cps and then decreases in proportion to $1/\nu$ with further increase in frequency. By combining this estimate with the forementioned description of the solar spectrum and evaluating the resulting expression, an approximate idea of the dependence of photoelectric emission on the satellite potential can be determined.

The photoelectric work functions of engineering materials range from about 3 to 4.5 v, corresponding to threshold frequencies of from 0.7×10^{15} sec^{-1} to 1.1×10^{15} sec^{-1}. Using a threshold frequency of 10^{15} sec^{-1} ($\varphi_w = 4.1$ v), it is found that the total photoemission current for $\varphi_s < 0$ is made up of 4 more or less equal contributions, corresponding to solar emission above 3×10^{15} sec^{-1}, emission between 2.14×10^{15} and 3×10^{15} sec^{-1} excluding Lyman α, the Lyman α emission, and emission below 2.14×10^{15} sec^{-1}.

Now, it is clear that for $\varphi_s > 0$ the positive ion current is always negligible in comparison with the electron current, and hence the satellite potential for the case where $\varphi_s > 0$ can be found from a current balance of the form

$$n_e e \bar{c}_e \left(1 - \frac{e\varphi_s}{kT}\right) = \frac{I_{ph_m}}{\pi R^2} f(\varphi_w + \varphi_s) \tag{4b}$$

where I_{ph_m} is the total photoelectric current for $\varphi_s \leq 0$ and $f(\varphi_w + \varphi_s) \leq 1$. As in the case of negative potentials, the term in parentheses on the left-hand side of Eq. (4b) is only valid when the satellite potential is completely unshielded. To estimate the maximum positive potential, this term should be neglected. Since the ion current has been neglected, Eq. (4b) is valid regardless of whether $v_s > \bar{c}_i$.

The measured values of photoelectric current density $I_{ph_m}/\pi R^2$ are less than 10^{-4} amp/m², and hence it can be seen from Table 1 and Eq. (4b) that so long as $n_e > 4 \times 10^9$/m³, positive satellite potentials would not be expected to occur. Since even in interplanetary space the electron density is around 10^8/m³, it is hard to predict positive satellite potentials of more than a few volts. The forementioned calculation overestimates the potential in any case, since the experimental results indicate that the mean energy of emitted photoelectrons is $\frac{1}{2}$ to $\frac{2}{3}$ of the maximum value given by Einstein's equation, and since it is probable that the incident electron current I_e does increase with increasing positive potential.

Although the calculations are for somewhat different circumstances, it is of interest to compare the forementioned with the results obtained by Kurt and Moroz.[19] They calculate the potential of conducting sphere in interplanetary space, considering the effects of ion and electron collection, photoelectric emission, and the collection of high energy electrons and protons in the radiation belts with the consequent emission of secondary electrons. In calculating the electron and ion currents, the approximate shielding distances given by Mott-Smith and Langmuir[16] are used, the motion of the sphere being neglected. The photoelectric current is obtained from a semigraphical analysis, the dependence on potential being roughly equivalent to that just given. For $T \leq 10^4$°K, charged particle density $n_e = n_i \geq 10^7/\text{m}^3$, and a theoretical photoelectric current density of 0.25×10^{-4} amp/m² (maximum value, for $\varphi \leq 0$), the sphere potentials outside the radiation belts are calculated as $-3.2 \leq \varphi \leq +4$ v. In the radiation belts, negative potentials as high as 2×10^4 v are estimated, although it is stated that the estimates are very rough. It is interesting to note that, contrary to the predictions of Chopra,[5] high positive potentials are not expected to occur. The experimental results to be described later indicate that an unrealistically high value of photoelectric emission current is used in Ref. 5.

It should be noted that in none of the methods just discussed is the complete problem of the potential assumed by the satellite treated. The ion and electron currents to the body are influenced by the body potential, since the effective collection area for ions or electrons is a function of potential. However, the body potential also serves to alter the ion or electron concentration in the neighborhood of the body, which will in itself affect the collection currents, and hence the body potential. An accurate solution to the problem must involve a calculation producing ion and electron currents, satellite potential, and charged particle densities around the satellite which are mutually consistent. A complete solution is not required for the purposes of this paper, since the limiting behavior has been determined. All of the forementioned approximate solutions give similar results, suggesting that the correction to be obtained from such a solution would be small.

Satellite Shielding

The shielding of a rapidly moving sphere of arbitrary potential has been considered by Davis and Harris[20] for the case where $\bar{c}_i \ll v_s \ll \bar{c}_e$. The solution is performed by an iterative procedure, whereby Poisson's equation is solved around the satellite for an assumed ion distribution, new ion trajectories are calculated, giving a modified ion distribution, Poisson's equation is again solved, and the procedure repeated until a self-consistent solution is obtained. Although particle-particle interactions are not included, the collective effects of the ion distribution are contained in Poisson's equation. The electron distribution is assumed to be given by $n_e = n_{e_\infty} e^{-(e\varphi/kT)}$. Solutions were obtained only for somewhat higher potentials than are now expected to occur, but the qualitative results are of interest. A region of reduced ion density is found to occur behind the satellite, where ions have effectively been swept out. Due to the focusing effect of the satellite potential on deflected ions, a small region of high ion density is found just behind the satellite on the axis.

Davis and Harris also plot ion densities and net charge densities in the vicinity of the sphere. It is found that although ions are effectively swept out by the sphere, the focusing effect of the sphere potential on the ions that do not actually collide with the sphere causes an appreciable concentration of ions in the wake, and the satellite is completely surrounded by an ion sheath.

It should be noted that in Davis and Harris' calculations the thermal velocity of ions has been completely neglected, and hence the wake is populated only by ions deflected by the satellite potential. The electron density is calculated by assuming that the electrons move in the combined field of the ions and the satellite, and solving Poisson's equation in the form

$$\nabla^2\varphi = \frac{e}{\epsilon_0} n_{e_\infty} \left(\frac{n_i}{n_{i_\infty}} - e^{-(e\varphi/kT)}\right) \tag{5}$$

where $n_{e_\infty} = n_{i_\infty}$ is the charged particle density in the undisturbed plasma and $\epsilon_0 = 8.85 \times 10^{-12}$ (MKS).

In connection with the numerical results obtained by Davis and Harris, it is of interest to consider the work of a number of authors on the nature of the flow field and potential distribution around a satellite moving in the ionosphere. If the change in ion density in the vicinity of the satellite and the lack of spherical symmetry are neglected, the potential distribution can be found directly from a solution of Eq. (5), where the first term in the parentheses is now equal to unity. This problem is treated by Jastrow and Pearse[9] and Chang and Smith,[11] who obtained similar results. However, the changes in ion density and the lack of spherical symmetry found by Davis and Harris leave the accuracy of these results open to question.

In a recently published paper, Hohl and Wood[14] present a set of numerical calculations based on an approach similar to that taken by Jastrow and Pearse, but including the potential gradient induced by a magnetic field normal to the velocity vector. Changes in ion density in the vicinity of the satellite are neglected, as is the existence of a wake region. Thus the spherical symmetry assumed by Jastrow and Pearse is replaced by rotational symmetry about an axis mutually normal to the velocity vector and the magnetic field. Hohl and Wood point out that a correct description of the electron density in the vicinity of the satellite must include a consideration of the absorption of electrons at the satellite surface. If the distance in which the satellite potential is shielded is small compared to the satellite diameter, the problem can be treated as plane and the electron density for $\varphi_s < 0$ is given by

$$n_e = n_{e_\infty} e^{-(e\varphi/kT)} \left[\frac{1}{2} + \frac{1}{2}\,\text{erf}\left(\frac{e(\varphi_s - \varphi)}{kT}\right)^{1/2}\right]$$

Thus the form of Poisson's equation solved in Ref. 14 is

$$\nabla^2\varphi = \frac{e}{\epsilon_0} n_{e_\infty} \left\{1 - e^{-(e\varphi/kT)}\left[\frac{1}{2} + \frac{1}{2}\,\text{erf} \times \left(\frac{e(\varphi_s - \varphi)}{kT}\right)^{1/2}\right]\right\}$$

Numerical calculations of the potential distribution and the resultant ion trajectories are presented for a 4 meter diameter sphere in a helium atmosphere. As would be expected, the results are similar to those obtained by Jastrow and Pearse.

The satellite potential calculated in Ref. 14 includes the effect of increased collection area for ions. The midpoint potential on which the gradient of Bv_s volts/meter is superimposed is found to be -0.396 v. The most negative average potential predicted by Eq. (3) is -0.388 v. The slight disagreement is due to the fact that the exponential dependence of electron current on potential, combined with the existence of a potential gradient, tends to increase the average negative potential somewhat compared to the zero magnetic field case.

In the works of Gurevich[21] and Dolph and Weil[22] the effect of the satellite charge on the ion motion is neglected, and the ion density distribution is found from a solution of the collision free Boltzmann equation. The distribution of ions is thus identical to the distribution of neutral particles, and the wake is populated by ions which diffuse into this region. The results of the two computations are in as good agreement as could be expected, since somewhat different approximations are used. A region of reduced density extends for a considerable distance behind the sphere, the density at 20 sphere diameters being about 90% of the freestream value.

Gurevich presents an approximate solution of Eq. (5), whereby a region of negative potential is found to exist for some distance behind the sphere. This negative potential serves to reduce the electron density in the wake, and Gurevich finds that to terms of order $(\lambda_D/R)^2$ plasma neutrality exists within the wake. λ_D is the debye length, and is of the order of a few centimeters in the upper atmosphere. Gurevich does not present accurate results for the immediate vicinity of the satellite. The inclusion of the electrostatic deflection of ions in the calculations would serve to reduce both the extent of the wake and the magnitude of the negative potential in the wake region.

In all the results concerning the wake just mentioned the effect of the earth's magnetic field is neglected. Gurevich extends his analysis of the wake region to include a constant magnetic field. He finds that where motion of the body perpendicular to the field lines is present, the ion density oscillates somewhat in the wake, and the return to the freestream value is more gradual than in the absence of a magnetic field.

In the present analysis it has been assumed that all ions incident on the satellite are neutralized and reflected as neutral particles. On the basis of the experience with probes in gas discharges, this would appear to be a reasonable assumption. However, no directly applicable experimental or theoretical results are available for conditions corresponding to those existing in the upper atmosphere, and hence the possibility of ions being reflected as ions should perhaps be considered. By means of a series of somewhat inconsistent plausibility arguments Chopra[23] develops a picture of the flow field which includes a belt of orbiting ionized particles a short distance from the sphere, and a longitudinal ionized column moving ahead of the sphere.§ In a more careful analysis, which includes the effects of coulomb interactions between the incoming and reflected ions, Bird[24] shows that for a disc moving perpendicular to its flat face, a certain percentage of the reflected particles will re-impinge on the surface. A parallel magnetic field greatly enhances this effect.

As can be easily seen from the derivation of Eq. (3), a decrease in the effective ion current to the satellite causes an increase in the negative potential assumed by the satellite. As will be seen, the agreement of the small amount of available experimental data with the theoretical predictions assuming complete neutralization suggests that most incident ions are neutralized on the satellite surface.

Another phenomena which has not been considered is that of secondary emission from the satellite surface. Again experience with gas discharges suggests that this effect is not significant, but again there are no directly applicable experimental or theoretical results available. The effect of secondary emission on the satellite potential is the same as that of photoelectric emission, and if any data were available a similar treatment could be used.

Experimental Results

An experimental investigation of collection currents and equilibrium potentials is reported in Ref. 25. On the basis of the experimental data, some qualitative evaluation of the just mentioned theories is possible. The data were taken at an altitude of 1000 km on the Explorer VIII Satellite. The ion current to a small collector on the satellite was found to be unaffected by the relative position of the earth's magnetic field. The measured ion current is consistent with Davis and Harris' picture of ion trajectories, and the assumption that ions are incident only on the front surface of the satellite is, in fact, a good one.

The average satellite potential was found to be ~ -0.15 v, and, as predicted by Beard and Johnson,[10] there was a potential gradient in the $\mathbf{v} \times \mathbf{B}$ direction. The electron current was concentrated toward the positive end of the satellite. However, contrary to the assumption made by Beard and Johnson, the electron current was not confined to directions parallel to the magnetic field lines, but was present also on surfaces perpendicular to the field lines. Also, contrary to the predictions of Chang and Smith,[11] there appears to have been very little reduction in the electron current on the back side of the satellite (opposite the velocity vector). Thus a qualitative description of the experimental results shows ions incident from the direction of the velocity vector only, and electrons incident on all surfaces of the satellite, but concentrated toward that end of the satellite which has the most positive potential. The electron current on the back surface of the satellite was $\sim$0.75 of that on the front surface.

The experimental measurement of photoemission makes it possible to compare the observed satellite potential with the values predicted from Eqs. (3) and (4). The relevant quantities taken from Ref. 25 are $v_s = 7.4 \times 10^3$ m/sec, $T = 1100$°K, $n_i = n_e = 1.3 \times 10^{10}$/m³, and $i_{\text{photon}}/\pi R^2 \approx 0.4 \times 10^{-4}$ amp/m². Using Eq. (3), the values for the satellite potential range from -0.24 to -0.32 v. The values obtained from Eq. (4a) range from -0.12 to -0.20 v. In view of the uncertainty regarding satellite shielding and the approximate nature of the estimate of i_{photon} (the Explorer VIII satellite was not spherical) the agreement between theory and experiment is felt to be quite good.‖

There is considerable variation in the estimates of electron density at high altitudes, but, as is pointed out in Ref. 25, the satellite would certainly be expected to be positive during the day at altitudes above 4000 km. Positive potentials could be predicted at lower altitudes. It might be noted that at altitudes where positive potentials are likely, the ion current to the satellite has a negligible effect on the satellite potential. On the other hand, at altitudes where photoemission is negligible the satellite potential depends only on the ion current and the electron temperature. During periods when the satellite is not exposed to solar radiation, its potential will, of course, be negative.

A model for the sheath surrounding the charged satellite similar to that suggested in Ref. 12 is proposed in Ref. 25. It is concluded that since no ion current was measured on the back surface of the satellite an electron sheath must exist at that point. This conclusion seems difficult to support, as does the idea that this negatively charged area must be surrounded by an ion sheath. As can be seen from the particle trajectories calculated by Davis and Harris, ions are certainly present in the area behind the satellite. It is only that their directed velocities are such that they do not strike the satellite. These ions still make a definite contribution to the net charge distribution in the vicinity of the satellite. If over-all charge neutrality is assumed, the presence of an appreciable electron current at the back of the satellite could be taken as an indication that an appreciable number of ions are also present in that area. This conclusion is consistent with the calculations of Davis and Harris and with the results of Gurevich.

Charged Particle Drag—Direct Collisions

The charging of a satellite has been treated in considerable detail in the preceding section. This detail is felt to be justified, since the drag due to momentum exchange with charged particles is directly related to the number of charged particles which strike or are deflected by the body, which in turn depends on the extent to which the charge on the body is shielded from the surrounding plasma. Consider first the drag due to direct impact of particles on the satellite. This problem has been considered in Refs. 9–11, 14, and 20.

§ In his two recent papers on this problem, Chopra[5, 23] presents a number of contradictory intuitive pictures of the charged particle field around a satellite. He makes no attempt to reconcile these contradictions, and the absence of derivations or calculations makes it difficult to evaluate the significance of his work.

‖ As is discussed in Refs. 26 and 27, the relatively high negative potentials reported for Sputnik III[28] are not felt to be accurate.

Within the scope of the so-called hypersonic approximation, where $v_s \gg \bar{c}_i$, the drag on a sphere due to incident ions (or neutral particles) can be expressed by

$$F_i = \pi m_i n_i v_s^2 b_i^2 \qquad (6)$$

where b_i is the effective collection radius for ions. The drag due to neutral particles is simply

$$F_n = \pi m_n n v_s^2 R^2 \qquad (7)$$

where $m_i \approx m_n$. In Ref. 2 it is shown that the estimate given by Eq. (7) is less than 10% low at a speed ratio $v_s/\bar{c}$ of 3, and is still less than 25% low at a speed ratio of 2. Thus the drag estimates calculated here should be valid so long as the principal atmospheric constituent is not hydrogen.

As has been seen in the previous discussion, the estimate of the effective collection radius given by Chang and Smith[11] can be taken as an upper limit on b_i, and thus can be effectively used to determine where the contribution of direct impacts with charged particles to the satellite drag may be neglected. Thus one writes

$$b_i = R\left(1 + \frac{2e\varphi_s}{m_i v_s^2}\right)^{1/2} \qquad (8)$$

as the upper bound. (For positive satellite potentials, Eq. (8) is still valid, and now $b_i < R$. The neglect of charged particles will now cause the drag to be somewhat overestimated.) Any shielding of the satellite potential will serve to reduce the value of b_i given in Eq. (8). A comparison with the approximate shielding distance calculated by Jastrow and Pearse[9] is not possible, since they solve only a spherically symmetric form of Poisson's equation, and neglect the effect of the directed velocity of the ions on the ion charge distribution. Beard and Johnson[10] use the result obtained in Ref. 9 without change. For the conditions used by Hohl and Wood,[14] Eq. (8) gives $b_i = 1.18\ R$, whereas their calculation yields $b_i = 1.05\,R$.

The ratio of the incident charged particle drag to the neutral particle drag is given by

$$\frac{F_i}{F_n} = \frac{n_i}{n}\left(1 + \frac{2e\varphi_s}{m_i v_s^2}\right) \qquad (9)$$

Equation (9) is only felt to be valid below the hydrogen region. Since, as one has seen, the satellite potential is never more than about 0.75 v negative, the term in parentheses is always less than 2. Thus the drag due to incident charged particles is only significant in those regions where the charged particle density is significant. It can be seen from Table 1 that neglect of the incident charged particles is certainly justified at lower altitudes, and even at 3000 km the drag force will be underestimated by at most 20%.

Rather than considering Eq. (9), it is probably more reasonable to calculate the error which would result from regarding all particles as neutral, and thus neglecting the increase in drag which occurs since some particles are charged. In this case, the error in the estimate of the total drag due to incident momentum transfer will always be less than 10% when the percentage of total particles which are charged is less than 10%.

The drag due to re-emitted particles has not yet been considered. Since it is assumed that all incident ions which strike the surface are re-emitted as neutral atoms, the estimates made by Schamberg[2] for neutral particles can be used in the present analysis. Following the approach of the preceding paragraph, consider only the errors which could result from neglecting the fact that some of the re-emitted neutral particles arrived at the surface as ions. As has been pointed out by Jastrow and Pearse,[9] the ions which arrive at the surface have been accelerated through an electric field, and since these ions leave the surface as neutral particles, a similar deceleration does not take place. If little or no accommodation to the surface temperature takes place, the recombined ions leave the surface with higher velocities than the particles which were always neutral.

Schamberg shows that for specular reflection the drag on a sphere is unaffected by the re-emitted molecules, and is always given correctly by Eq. (7). Thus, to estimate the maximum effect of the ions, consider the case of diffuse re-emission without any surface accommodation. Following Schamberg, the contribution of re-emitted particles to the drag for this limiting case is given by

$$F_r = nmv_s^2\pi R^2\left(\frac{4}{9}\,\frac{v_r}{v_s}\right) \qquad (10)$$

where v_r is the velocity of re-emission, and for neutral particles $v_r = v_s$ in our limiting case. In the case of incident ions, v_r is given by $v_r = v_s + (2e\varphi_s/m_i)^{1/2}$, and thus the maximum contribution of re-emitted recombined ions to the drag is, from (6), (8), and (10)

$$F_{ri} = n_i m_i v_s^2 \pi R^2\left(1 + \frac{2e\varphi_s}{m_i v_s^2}\right)\left\{\frac{4}{9}\left[1 + \left(\frac{2e\varphi_s}{m_i v_s^2}\right)^{1/2}\right]\right\} \qquad (11)$$

It should be noted that the additional collection area for ions has been included in Eq. (11).

Since the maximum value of $(2e\varphi_s/m_i v_s^2)^{1/2}$ is slightly less than 1, Eq. (11) yields a maximum drag contribution which is about 4 times that of Eq. (7), for similar freestream densities. Any accommodation of the particles to the surface temperature will rapidly reduce this value.

As before, the meaningful quantity is the additional drag which results from the fact that a certain percentage of the total particles are ionized rather than neutral. From Eqs. (6, 7, 10, and 11), and using the maximum possible value (below the hydrogen region) of $2e\varphi_s/m_i v_s^2 = 1$, it can be seen that if 10% of the particles are ionized, the neglect of this ionization causes the total drag due to direct collisions with ions to be underestimated at most by 12%.

The momentum transfer due to collisions with electrons can be estimated by means of a very simple argument. Following the approach of Jastrow and Pearse,[9] make use of Epstein's expression for the drag on a sphere in free molecule flow,[29] for the case where $v_s \ll \bar{c}_e$

$$F_e = (4\pi/3)n_e m_e \bar{c}_e R^2 v_s \qquad (12)$$

Taking the ratio of Eq. (12) to Eq. (6), using $b_i \approx R$ and $n_i = n_e$, one has $F_e/F_i \approx m_e\bar{c}_e/m_i v_s$, indicating that the drag due to electron impacts is always less than 1% of that due to ion impacts. It should be noted that Eq. (12) probably overestimates the drag due to electrons, since most electrons are only deflected by the body and do not collide with it.

Charged Particle Drag—Noncolliding Deflected Particles

Having estimated the drag due to charged particles which collide with the satellite, one must now consider the effects of what is called "dynamic friction." Here one refers to the momentum exchange which takes place between the charged sphere and those charged particles which are deflected by the coulomb forces, but do not actually strike the sphere. In other words, consider ions with impact parameter $b > b_i$, which pass through that region where the satellite potential is not yet completely screened. This problem has been considered in Refs. 9, 11, 14, 20, and 30.

Jastrow and Pearse[9] calculate the effect of deflected ions using their calculated shielding distance, and conclude that the total contribution to the drag is negligible. A similar result is obtained by Hohl and Wood.[14] However, in view of the inaccuracies which exist in their shielding models, this result should not be regarded as conclusive.

Chopra and Singer[30] attempt to estimate the dynamic friction through an application of Spitzer's[4] calculation of the slowing down of ions by charged field particles. Briefly, the approach taken by Spitzer involves a modification of the theory advanced by Chandrasekhar[31] for dynamic friction due to gravitational forces to apply to the case of coulomb forces. The resulting drag is given by[4]

$$F = \frac{4\pi n_i e^2 (Ze)^2}{m_i v_s^2} \ln\left(\frac{b_m}{b_i}\right) \tag{13}$$

where b_m is the impact parameter beyond which the ion motion is unaffected by the satellite potential and Ze is the total charge on the satellite. If the satellite is regarded as unshielded, $Ze = 4\pi\epsilon_0 R\varphi_s$ where $\epsilon_0 = 8.85 \times 10^{-12}$. Any shielding of the satellite will increase this value. However, as is pointed out by both Chandrasekhar and Spitzer, Eq. (13) is obtained under the restriction that $\ln(b_m/b_i) \gg 1$, and in fact $\ln(b_m/b_i)$ represents the ratio of the dominant to nondominant terms in the more complicated expression developed by Chandrasekhar. It seems clear that for normal sized satellites $\ln(b_m/b_i)$ is probably less than one. In place of $\ln(b_m/b_i)$, Chopra and Singer used $\ln(\lambda_D/p_0)$, where p_0 is the impact parameter for 90° deflection in a collision between singly charged ions and λ_D the debye length. Thus, the rather large drag forces calculated in Ref. 30 have no significance.#

Chang and Smith[11] obtain Eq. (13) from a simplified treatment in which only the momentum acquired by deflected particles in a direction normal to the satellite velocity is considered. However, no conclusions as to the appropriate magnitude of b_m are possible without a calculation of the actual charge distribution around the satellite.

The somewhat controversial paper by Wyatt[32] should perhaps be mentioned at this point. This work is discussed at length elsewhere in the literature,[33-36] where it is pointed out that the lack of a physically realistic basis for the analysis has produced misleading results.

Rand[37] considers a circular disk of radius $R \gg \lambda_D$ moving in a direction perpendicular to its surface. He calculates the drag due to deflected particles in a manner somewhat different from that just given. In addition to assuming that complete shielding takes place within a debye length of the disk edge, he makes use of linearizations which require that the inequality $e\varphi \ll kT$ be satisfied. For typical conditions $e/kT \approx 10/\text{v}$, and hence it must be concluded that if the satellite carries any significant charge, and hence can be expected to have significant drag due to deflected particles, the results obtained by Rand are not applicable.

Some conclusions as to the effect of deflected particles can be obtained from the numerical calculations of Davis and Harris.[20] They compute the drag due to deflected particles, and then modify the effective impact parameter b_i in such a manner that the total drag due to charged particles is given by Eq. (6). The effect of the re-emitted particles is neglected. The satellite velocities used in the calculations are appropriate to the region below 1000 km. Results are presented for spheres having radii of 10 and 25 debye lengths, at values of the quantity $2e\varphi_s/m_i v_s^2$ between 0.6 and 6.** The maximum debye length below 1000 km is about 3 cm. The results show that for the 25 debye length sphere radius, the charged particle drag calculated using b_i from Eq. (8) is always greater than the value given by the numerical calculation. For the smaller sphere, Eq. (8) underestimates the charged particle drag by at most 15%. The amount by which b_i is in error seems to increase with decreasing potential, implying that the satellite is more effectively shielded at higher potentials. However, in view of the fact that at values of $2e\varphi_s/m_i v_s^2$, less than one the charged particle drag is already negligible; and since Davis and Harris show that the drag decreases with decreasing potential, it seems reasonable to conclude that, although Eq. (8) does not correctly give the total charged particle drag for small spheres at small potentials, its use to justify the neglect of the fact that some atmospheric particles are ionized is always permissible. Thus, a determination of the effective shielding distance b_m is not required. Note that for objects much smaller than the debye length, the major contribution to the drag comes from noncolliding deflected particles. This subject is outside the scope of the present article, and is treated elsewhere in the literature.

In a more recent paper Chopra[23] describes λ_D/p_0 as "a more plausible parameter." However, no arguments to support this description are presented, and it is difficult to see why quantities derived for a singly charged ion are plausible for a multiply charged body.

** Contrary to the statement made by Hohl and Wood,[14] their results are actually for conditions comparable to those used by Davis and Harris.[20] Although a lower potential is used in Ref. 14, lighter ions are considered. The average value of $2e\varphi_s/m_i v_s^2$ used by Hohl and Wood is 0.39.

Induced Drag

Since the satellite has been assumed to be made of a conducting material, the motion of this conductor across the earth's magnetic field lines will cause a current flow, and a resultant force on the conductor. The magnitude of this force can be obtained, at least in theory, in terms of the electrical conductivities of the satellite and the surrounding plasma. This problem is treated by Jefimenko.[38] However, as is pointed out in Ref. 38, the use of the usual macroscopic electrical conductivity of the surrounding plasma is not correct, since the dimensions of the satellite are much smaller than the mean free paths in the plasma.

Since the current flow to the satellite already has been considered, an estimate of the induced drag can be obtained by means of a very simple argument. The total ion current to the satellite, neglecting the increase in the effective collection area due to the potential, is simply $I_i = -en_i v_s \pi R^2$. Now, as is pointed out by Beard and Johnson, the motion of the satellite through the magnetic field produces a potential gradient, which tends to concentrate the electron current toward the positive end of the satellite. Neglecting the effects of photoelectric emission, and regarding the ion current as being at the negative end of the satellite, the total current flow is just equal to the ion current. Thus, since the force on a conductor moving perpendicular to a magnetic field is simply $F = IlB$, where l is the effective conductor length, the induced drag is given by $F = -2en_i v_s \pi R^3 B$. The effective length has been taken as $2R$. This expression is obtained by Jefimenko from a somewhat different argument, in which he regards the satellite as an electric dipole.

Since the ion current is actually distributed more or less uniformly over the satellite surface, the effective current is about half the total current, and thus the induced drag expression just obtained should be divided by two, to yield $F = -en_i v_s \pi R^3 B$. This is the expression obtained by Beard and Johnson for large satellites, except for a numerical factor of 0.85. To include the effect of potential on the ion current, one makes use of the fact that, since the potential gradient induced in the satellite is Bv_s/meter, and one end of a large satellite is essentially at the plasma potential, the average satellite potential is $\varphi_0 = Bv_s R$. Thus, the induced drag is given by

$$F = -en_i v_s \pi R^3 B \left(1 - \frac{eBv_s R}{m_i v_s^2}\right) \tag{14}$$

In the region where this formulation is valid (below the hydrogen level) the effect of photoelectric emission may serve to increase the effective current, and thus the induced drag, by as much as a factor of 10.

In the more careful analysis of this problem presented by Hohl and Wood,[14] which appeared after the bulk of the present article was completed, the satellite is divided into sections, and the current flow to each section is considered separately; although the effect of potential on the ion current is neglected. Photoelectric emission is not considered, and the resulting drag is somewhat smaller than that predicted by Eq. (14). Hohl and Wood also include a discussion of the effect of the internal resistance of the satellite, which has been neglected here, and show that this is in fact negligible.

It is of interest to compare the induced drag with the incident charged particle drag, as given by Eq. (6). Taking the ratio of Eq. (14) to Eq. (6), using b_i as defined in Eq. (8), with $\varphi = \varphi_0 = Bv_sR$, one has

$$\frac{F_{\text{induced}}}{F_{\text{charge}}} = -\frac{eRB}{n_iv_s} \tag{15}$$

Taking the magnetic field strength from Ref. 3 as $B \approx 3 \times 10^{-5}$ webers/m^2, and using the data of Table 1, the maximum value of Eq. (15) is about 0.1 R, where R is in meters. Thus, for very large satellites, the induced drag may be significant, and, at extremely high altitudes, where the photoelectric current is greater than the incident ion current (note that the emission of a photoelectron is equivalent to the collection of an ion) the induced drag probably exceeds the neutral particle drag. At 3000 km the photoemission current is estimated to be about 20 times the incident ion current. It seems clear that, if the drag in the helium region is to be calculated, the induced drag must be considered in detail. Below this region the effect is negligible.††

Wave Drag

As is explained by Lighthill,[39] the deflection of charged particles in a plasma actually takes place in a large number of extremely small steps, and hence even in the case when the free path on the basis of 90° deflection is large, the actual particle momentum is being changed in a more or less continuous manner, and hence it is clearly possible that relatively small bodies can excite some sort of collective plasma motion. The question of collective plasma motion (plasma waves or plasma oscillations) and the associated momentum loss by the body exciting this motion is directly related to the problem of calculating the potential and electric field distribution around the body, and of determining the resultant charged particle deflections. It would appear that if the satellite moves into an undisturbed plasma, all contributions to the drag force on the satellite due to any interaction with that plasma have been considered in the preceding sections. It is of course possible that the perturbations of the plasma produced by the satellite could in turn disturb the plasma at the satellite, thus altering the forces on the satellite. Any waves which do not alter the plasma at the satellite will serve only to dissipate energy, the transfer of which from the satellite to the plasma has already been considered. In the final analysis, the question seems to be whether or not it is correct to treat the potential and charge distribution around the satellite as a time independent problem. (Similar conclusions regarding wave drag are presented by Hohl and Wood.[14])

The calculations performed by Kraus and Watson[40] are actually concerned with the time independent problem, and hence, under the classification system used in this paper, should correctly be considered in the section on drag due to noncolliding deflected particles. Their analysis is restricted to the case where $\bar{c}_i \ll v_s \ll \bar{c}_e$, and they treat only bodies very much smaller than the debye length. The maximum debye length in that portion of the upper atmosphere in which the ionic thermal velocity can be neglected is about 7 cm. As would be expected, their results differ very little from those obtained by Spitzer[4] [see Eq. (13)]. Greifinger[41] extends Kraus and Watson's results to include the effects of a magnetic field. However, in a frame of reference moving with the body the problem is still independent of time. The results indicate that the presence of a magnetic field modifies the drag for zero magnetic field; however, the restriction on body size imposed by Kraus and Watson is retained, and hence the significance of these results for satellites of realistic size cannot be determined.

No treatment of the time dependent problem seems to exist at the present time, and hence the possible contribution of collective plasma motion to the drag on a satellite cannot be estimated. It is possible that a continuum approach, such as that suggested by Lighthill[39] may provide a more straightforward method of solving this problem.

Conclusions

The analysis of the preceding sections has shown that, apart from the possible effects of plasma waves or plasma oscillations, in the calculation of the drag on a satellite large compared to the debye length the fact that some atmospheric particles are charged can be neglected below the altitude at which helium becomes the principal atmospheric constituent. In the helium region the induced drag on a large satellite may be significant, particularly in cases where the photoelectric emission current exceeds the incident ion current. A means of estimating this effect has been given. The methods of calculation used in the preceding sections are not applicable to the region where hydrogen is the principal atmospheric constituent.

References

[1] Schaaf, S. A., "Aerodynamics of satellites," *Aerodynamics of the Upper Atmosphere*, edited by D. J. Masson (Rand Corp., 1959), pp. 1-1–1-14.

[2] Schamberg, R., "Analytic representation of surface interaction for free molecule flow with application to drag of various bodies," *Aerodynamics of the Upper Atmosphere*, edited by D. J. Masson (Rand Corp., 1959), pp. 12-1–12-41.

[3] Johnson, F. S., ed., *Satellite Environment Handbook* (Stanford University Press, Stanford, Calif., 1961), Chaps. 1 and 2.

[4] Spitzer, L., Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), Chap. 5.

[5] Chopra, K. P., "Interactions of rapidly moving bodies in terrestrial atmosphere," Rev. Mod. Phys. **33**, 153–189 (1961).

[6] Jastrow, R., "Results of experiments in space," J. Aerospace Sci. **29**, 377–388 (1962).

[7] Dalgarno, A., McElroy, M. B., and Moffett, R. J., "Electron temperatures in the ionosphere," Planet. Space Sci. **11**, 463–484 (1963).

[8] Bourdeau, R. E., Whipple, E. C., Jr., Donley, J. L., and Bauer, S. J., "Experimental evidence for the presence of helium ions in the upper atmosphere based on Explorer VIII Satellite data," J. Geophys. Res. **67**, 467–475 (1962).

[9] Jastrow, R. and Pearse, C. A., "Atmospheric drag on the satellite," J. Geophys. Res. **62**, 3, 413–423 (1957).

[10] Beard, D. B. and Johnson, F. S., "Charge and magnetic field interaction with satellites," J. Geophys. Res. **65**, 1–7 (1960).

[11] Chang, H. H. C. and Smith, M. C., "On the drag of a spherical satellite moving in a partially ionized atmosphere," J. Brit. Interplanet. Soc. **17**, 199–205 (1960).

[12] Gringauz, K. N. and Zelikman, M. Kh., "Measurement of the concentration of positive ions along the orbit of an artificial satellite," Usp. Fiz. Nauk **63**, 239–252 (1957); transl. in, *The Russian Literature of Satellites*, Part II (International Physical Index, New York, 1958), pp. 133–147.

[13] Lehnert, B., "Electrodynamic effects connected with the motion of a satellite of the earth," Tellus 3, 408–409 (1956).

[14] Hohl, F. and Wood, G. P., "The electrostatic and electro-

†† The extremely large induced drags calculated by Chopra[5] are due to the use of the macroscopic plasma electrical conductivity. Chopra claims that the use of the macroscopic conductivity is justified by the fact that this conductivity is only logarithmically dependent on the electron density. The reasoning behind this statement is difficult to understand, since a correct macroscopic treatment must yield the same results as the microscopic approach, and the discrepancy between the two results is noted in Ref. 5.

magnetic drag forces on a spherical satellite in a rarefied partially ionized atmosphere," *Rarefied Gas Dynamics*, edited by J. Laurmann (Academic Press Inc., New York, 1963), pp. 45–64; a shortened version appears in NASA SP-25, pp. 33–41 (December 1962).

[15] Hanson, W. B., "Upper atmosphere helium ions," J. Geophys. Res. **67**, 183–188 (1962).

[16] Mott-Smith, H. M. and Langmuir, I., "The theory of collectors in gaseous discharges," Phys. Rev. **28**, 727–763 (1926).

[17] Heatley, A. H., "Collector theory for ions with Maxwellian and drift velocities," Phys. Rev. **52**, 235–238 (1937).

[18] Weissler, G. L., "Photo-ionization in gases and photoelectric emission from solids," *Handbuch der Physik XXI* (Springer-Verlag, Berlin, 1956), pp. 304–382.

[19] Kurt, P. G. and Moroz, V. I., "The potential of a metal sphere in interplanetary space," Iskusstvennye Sputniki Zemli **7**, 78–88 (1961); transl. in, Planet. Space Sci., **9**, 259–268 (1962).

[20] Davis, A. H. and Harris, I., "Interaction of a charged satellite with the ionosphere," *Rarefied Gas Dynamics*, edited by L. Talbot (Academic Press Inc., New York, 1961), pp. 691–699.

[21] Gurevich, A. V., "Perturbations in the ionosphere caused by a travelling body," Iskusstvennye Sputniki Zemli **7**, 101–124 (1961); transl. in, Planet. Space Sci. **9**, 321–344 (1962).

[22] Dolph, C. L. and Weil, H., "On the change in radar cross-section of a spherical satellite caused by a plasma sheath," Planet. Space Sci. **6**, 123–132 (1960).

[23] Chopra, K. P., "Review of electromagnetic effects on space vehicles," J. Astronaut. Sci. IX, 10–17 (1962).

[24] Bird, G. A., "The flow about a moving body in the upper ionosphere," J. Aerospace Sci. **29**, 808–814 (1960).

[25] Bordeau, R. E., Donley, J. L., Serbu, G. P., and Whipple, E. C., Jr., "Measurement of sheath currents and equilibrium potential on the Explorer VIII satellite," J. Astronaut. Sci. **8**, 65–73 (1961).

[26] Whipple, E. C., Jr., "The ion-trap results in 'Exploration of the upper atmosphere with the help of the third Soviet Sputnik,'" Proc. Inst. Radio Engrs. **47**, 2023–2024 (1959).

[27] Gringauz, K. I., Bezrukikh, V. V., and Ozerov, V. D., "Results of measurements of the concentration of positive ions in the atmosphere, using ion traps mounted on the third Soviet earth satellite," *Artificial Earth Satellites*, edited by L. V. Kurnosova (Plenum Press, Inc., New York, 1961), Vol. 6, pp. 77–121.

[28] Krassovsky, V. I., "Exploration of the upper atmosphere with the help of the third Soviet Sputnik," Proc. Inst. Radio Engrs. **47**, 289–296 (1959).

[29] Epstein, P. S., "On the resistance experienced by spheres in their motion through gases," Phys. Rev. **23**, 710–733 (1924).

[30] Chopra, K. P. and Singer, S. F., "Drag of a sphere moving in a conducting fluid in the presence of a magnetic field," *Proc. 1958 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1958), pp. 166–175.

[31] Chandrasekhar, S., "Dynamical friction," Astrophys. J. **97**, 255–273 (1943).

[32] Wyatt, P. J., "Induction drag on a large negatively charged satellite moving in a magnetic field free ionosphere," J. Geophys. Res. **65**, 1673–1678 (1960).

[33] Beard, D. B. and Johnson, F. S., "Comment on Wyatt's analysis of charge drag," J. Geophys. Res. **65**, 3491–3492 (1960).

[34] Licht, A. L., "The drag on a charged satellite," J. Geophys. Res. **65**, 3493 (1960).

[35] Wyatt, P. J., "Author's reply to the Beard-Johnson comments," J. Geophys. Res. **66**, 1578–1579 (1961).

[36] Beard, D. B. and Johnson, F. S., "Ionospheric limitations on attainable satellite potential," J. Geophys. Res. **66**, 4113–4122 (1961).

[37] Rand, S., "Wake of a satellite traversing the ionosphere," Phys. Fluids **3**, 265–273 (1960).

[38] Jefimenko, O., "Effect of the earth's magnetic field on the motion of an artificial satellite," Am. J. Phys. **27**, 344–348 (1959).

[39] Lighthill, M. J., "Note on waves through gases at pressures small compared with the magnetic pressure, with applications to upper atmosphere aerodynamics," J. Fluid Mech. **9**, 465–472 (1960).

[40] Kraus, L. and Watson, K. M., "Plasma motions induced by satellites in the ionosphere," Phys. Fluids **1**, 480–488 (1958).

[41] Greifinger, P. S., "Induced oscillations in a rarefied plasma in a magnetic field," *Aerodynamics of the Upper Atmosphere*, edited by D. J. Masson (Rand Corp., 1959), pp. 19-1–19-32.

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Integral Approach to an Approximate Analysis of Thrust Vector Control by Secondary Injection

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An approximate analysis of thrust vector control by secondary fluid injection is approached through the application of the integral form of the conservation laws and the equation of state for a mixture of gases. The thrust augmentation and the side force are expressed in terms of the flow conditions at the exit section of the nozzle and the problem is thus reduced to that of determining these conditions. In this sense the present approach is different from the usual one where the pressure distribution over the nozzle surface is the object of the analysis. Considering inert gases, the necessary equations are developed and the steps involved in obtaining a solution are discussed. An approximate formula for the side force, applicable under certain conditions of operation, is obtained. Results given by the formula are compared and found to be in agreement with appropriate experimental results.

Nomenclature

A = area
c = specific heat at constant pressure
d = diameter of nozzle exit
E = energy per unit length
$\mathbf{F}$ = force
δF_a = thrust augmentation
F_s = side force
h = enthalpy per unit mass
H = total or stagnation enthalpy per unit mass
$\mathbf{i}$ = unit vector in the direction of X axis
$\mathbf{j}$ = unit vector in the direction of Y axis

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